Vectors and Motion

2004	**C1.** The position of a power sledge on a frozen lake at time t seconds, relative to a rectangular coordinate system, is $$\mathbf{r}(t) = (2t^2 - t)\mathbf{i} - (3t + 1)\mathbf{j},$$ where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the x, y directions respectively and distances are measured in metres. Calculate the time at which the speed is $5\,\mathrm{m\,s^{-1}}$.	**4**
2009	**A4.** A particle moves in a straight line from the origin with initial velocity $u\mathbf{i}$ and uniform acceleration $a\mathbf{i}$, where $\mathbf{i}$ is a unit vector in a fixed direction. After time t, the velocity of the particle is $v\mathbf{i}$ and the displacement from the origin is $s\mathbf{i}$. (*a*) Using calculus, show that $v = u + at$ and that $s = ut + \frac{1}{2}at^2$. (*b*) Hence show that $v^2 = u^2 + 2as$.	**4** **2**
2005	**A5.** The velocity of an ice skater relative to a rectangular coordinate system with origin O, is given by $$\mathbf{v} = 3(t^2 - 4t + 2)\mathbf{i} + 4\mathbf{j},$$ where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the Ox and Oy directions, t seconds is the time and the speed is measured in $\mathrm{m\,s^{-1}}$. Initially the skater has position vector $-4\mathbf{j}$. (*a*) Find the time at which the acceleration is instantaneously equal to zero. (*b*) Calculate the distance of the skater from O when the acceleration is instantaneously equal to zero.	**2** **4**
2005	**A11.** A particle is projected horizontally from the origin, O, along the positive x-axis with initial speed $1\ \mathrm{m\,s^{-1}}$. The particle has acceleration $4(4x - 1)\mathbf{i}\ \mathrm{m\,s^{-2}}$, where x metres $(0 \leq x \leq \frac{1}{4})$ is the distance of the particle from O after time t seconds and $\mathbf{i}$ is a unit vector in the direction of the x-axis. (*a*) Show that the speed, $v\ \mathrm{m\,s^{-1}}$, of the particle is given by $$v = 1 - 4x.$$ (*b*) Hence show that $$x = \tfrac{1}{4}(1 - e^{-4t}).$$	**5** **5**
2006	**A1.** Relative to a rectangular coordinate system, the position of an ice skater at time t seconds is $$\mathbf{r}(t) = \left(\frac{1}{3}t^3 - 4t^2\right)\mathbf{i} - (2t^2 - 1)\mathbf{j},$$ where $\mathbf{i}$, $\mathbf{j}$ are the unit vectors in the x, y directions respectively and distances are measured in metres. Find the speed of the ice skater at the instant when the acceleration is parallel to the y-axis.	**5**

2007	**A1.** The position of a remote controlled model boat on a pond, relative to a rectangular coordinate system with origin O, is given by $$\mathbf{r} = (3t^2 - 12t + 5)\mathbf{i} + (4t - t^2)\mathbf{j},$$ where $\mathbf{i}$, $\mathbf{j}$ are unit vectors in the Ox and Oy directions respectively, t is time measured in seconds and distances are measured in metres. Calculate the distance of the boat from the origin O when it comes to instantaneous rest.	**4**
2008	**A1.** A particle has velocity $3t(2 - t)\mathbf{j}$ where $\mathbf{j}$ is the unit vector in the direction of motion. The time t is measured in seconds from the start of the motion and the displacement is measured in metres. Initially the particle is at the point with position vector $3\mathbf{j}$ relative to the origin O. Calculate the distance of the particle from O when the velocity is a maximum.	**4**
2009	**A3.** The position of a model boat P, relative to a rectangular coordinate system with origin O, is given by $$\mathbf{r}_P = t^2\mathbf{i} + 4t\mathbf{j}$$ where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Ox and Oy directions respectively, t is the time measured in seconds and distances are measured in metres. The acceleration of a second boat Q is given by $$\mathbf{a}_Q = 2\mathbf{i} + (4\pi \sin 2\pi t)\,\mathbf{j}.$$ Given that boat Q is initially at rest, find the first two times when the boats have the same velocity.	**5**
2013	**A1.** A particle is moving in a plane such that t seconds after the start of its motion, the velocity is given by $(3t\boldsymbol{i} + 5t^2\boldsymbol{j})\ \text{m s}^{-1}$. The particle is initially at the point $\left(\frac{1}{2}\boldsymbol{i} - 7\boldsymbol{j}\right)$ metres relative to a fixed origin O. Find the distance of the particle from O when $t = 3$.	**3**
2017	**3.** The velocity of a particle after t seconds of travel can be expressed as $\mathbf{v} = (3\sin 2t)\mathbf{i} + (\cos 2t - 3)\mathbf{j}\text{ms}^{-1}$ where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in horizontal and vertical directions respectively. Find the magnitude of the acceleration of the particle when $t = \dfrac{\pi}{6}$ seconds.	**4**